SAWTIMBER VOLUME-BASAL AREA RATIO EQUATIONS FOR RED PINE IN MICHIGAN

7.6" DIB

BY

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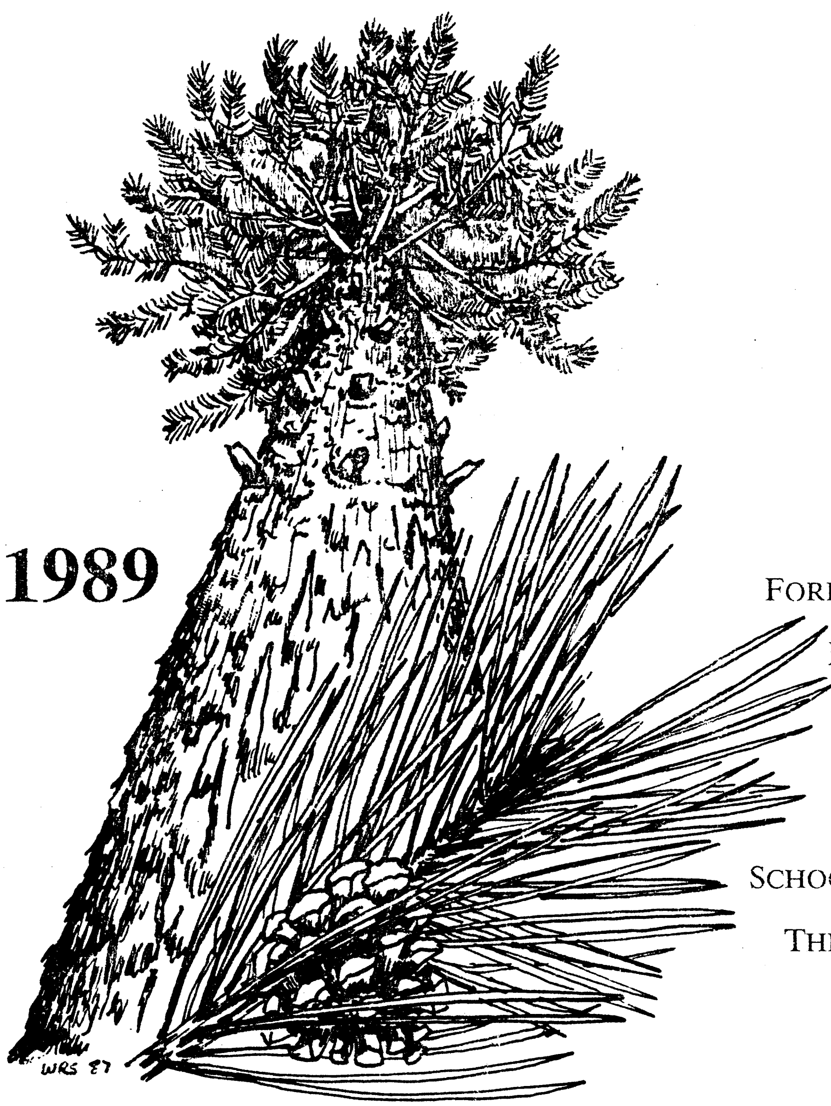

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MANAGEMENT SUMMARY

New cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot sawtimber volume-basal area ratio (VBAR in cu. ft./sq. ft. or bd. ft./sq. ft.) equations were developed for red pine in Michigan. Data used to develop these equations were collected from 27 red pine stands in Michigan (16 and 11 stands from the Upper and Lower Peninsulas, respectively).

VBAR multiple linear regression equations using diameter at breast height (DBH) and merchantable height independent variables yielded somewhat higher coefficients of determination (R^2), lower standard errors of estimate ($s_{y \cdot x}$), and more accuracy than VBAR equations using only height independent variables. However, an examination of prediction relative errors indicated that the height only VBAR equations are more than adequate for most situations. The new International 1/4-inch board-foot VBAR equation yields VBAR values relatively close to the values presently used by the Michigan Department of Natural Resources (MDNR). In general, estimates using the new VBAR equation would be no more than 4% more than those based on the current MDNR VBARs. Both sets of values are based on similar merchantability limits.

We recommend the use of the following sawtimber VBAR equations in most cruising situations for red pine:

1. Cubic-foot VBAR

$$\widehat{VBAR}_C = 10.897 + 3.605 \bullet H - 6.767 \bullet \frac{1}{H}$$

2. Doyle board-foot VBAR

$$\hat{VBAR}_D = -16.117 + 24.701 \bullet H + 9.533 \bullet \frac{1}{H}$$

3. International 1/4-inch board-foot VBAR

$$\hat{VBAR}_I = 35.253 + 26.090 \bullet H - 24.142 \bullet \frac{1}{H}$$

4. Scribner board-foot VBAR

$$\hat{VBAR}_S = 27.161 + 26.294 \bullet H - 19.309 \bullet \frac{1}{H}$$

In the above equations, H is the merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit.

Multiple linear regression equations were also developed to predict one type of VBAR from another for the four types of VBARs examined in this study.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

MICHIGAN DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT VOLUME-BASAL AREA RATIO EQUATIONS

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TITLE Sawtimber Volume-Basal Area Ratio Equations for Red Pine in Michigan

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Background

The Michigan Department of Natural Resources (MDNR) developed sawtimber volume basal area ratios (VBARs) in bd. ft. per sq. ft. to be used in estimating International 1/4-inch board-foot volume using prism cruising. MDNR Tally Sheet R 4145 was developed from these VBARs. These VBARs are composite and used for all species in Michigan without species adjustments being made.

Purpose

The purpose of this paper is to present new sawtimber cubic-foot and Doyle, International 1/4-inch, and Scribner VBAR equations and tables for red pine in Michigan.

Methods and Materials

Felled tree and/or standing tree measurements were made from a total of 3507 trees from 27 stands as follows:

1) 2341 trees from 16 stands in the Upper Peninsula (i.e., 5, 4, and 7 stands in the eastern, central, and western U.P., respectively), and

2) 1166 trees from 11 stands in the Lower Peninsula (i.e., 4, 4, and 3 stands from the northeast, northwest, and southern L.P., respectively).

Stands were selected from the six regions to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May - August, 1983-1985.

For the 27 stands, site index varied from 45 to 75, age varied from 26 to 105 years, basal area/acre varied from 90 to 225 sq. ft., average DBH varied from 6.7 to 17.7 in., average total height varied from 33.2 to 85.0 ft., and average merchantable height to an approximate 3.6-in. minimum top diameter varied from 2.0 to 8.4 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height (MH) to the nearest 100-in. stick to an approximate 3.6 in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5'), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987a). Fowler and Hussain (1987b) developed pulpwood, sawtimber, and residual pulpwood cubic-foot VBAR equations from the total data set described above.

Sawtimber trees were defined as trees that had at least one 100-in. stick with a minimum inside bark diameter no smaller than 7.6 inches. Sawtimber MH is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 inches. There was a total of 2,157 sawtimber trees.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into two pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the two volumes. For each 100 in. stick, cubic-foot and board-foot volumes were calculated using the following formulas:

Cubic-foot: $V = \frac{(B+b)L}{2}$ (Avery and Burkhart 1983)

Doyle: $V = 0.5D^2 - 4.0D + 8.0$ (Husch et al. 1982)

International 1/4-inch: $V=0.905(0.44D^2 - 1.2D - 0.30)$ (Husch et al. 1982)

Scribner: $V = 0.395D^2 - 0.99D - 2.15$ (Bruce and Schumacher 1950)

where

V = volume in cubic feet or board feet,

L = length of stick (100 in.) in ft.,

B = cross-sectional area inside bark of large end of the stick in sq. ft.,

b = cross-sectional area inside bark of small end of the stick in sq. ft., and

D = diameter of small end of stick inside bark in inches. See Avery and Burkhart (1983) and Husch et al. (1982) for detailed discussions of cubic-foot volumes and board-foot log rules.

Cubic-foot and the three board-foot volumes for each tree were determined by summing up the volumes of all sawtimber sticks to a 7.6-in. top diameter limit. Sawtimber VBARs were obtained for each tree by dividing cubic-foot and Doyle, International 1/4-inch, and Scribner volumes of the tree by the basal area in sq. ft. of the tree at 4.5 ft. from the ground.

VBAR was regressed on various forms of sawtimber MH and DBH using multiple linear regression.

Results

The data set used to develop the regression equations consisted of 1725 trees (≃80%) selected at random from the total of 2157 trees. Fowler and Hussain (1987b) found no significant differences between the pulpwood cubic-foot volume equations of the six regions, so the data from all stands were pooled before developing VBAR regression equations. All equations were based on 1725 trees with an average DBH = 12.2 in. (range: 7.9 to 23.9), average MH = 3.7 sticks (range: 1 to 8), average cubic-foot VBAR = 21.51 cu. ft./sq. ft. (range: 6.99 to 44.08), average Doyle board-foot VBAR = 78.07 bd. ft./sq. ft. (range: 9.21 to 200.96), average International 1/4-inch board-foot VBAR = 121.61 bd. ft./sq. ft. (range: 22.76 to 268.87), and average Scribner board-foot VBAR = 116.2 bd. ft./sq. ft. (range: 20.27 to 260.24).

Cubic-foot and Board-foot VBAR Prediction Equations

A comparison of various multiple linear regression equations based on goodness-of-fit and simplicity indicated that the following prediction equations compared favorably to all other equations examined for cubic-foot and the three board-foot VBARs:

1. Height independent variables

$$\hat{VBAR} = \hat{B}_0 + \hat{B}_1 H + \hat{B}_2 \frac{1}{H}$$

2. Height and diameter independent variables

$$\hat{VBAR} = \hat{B}_0 + \hat{B}_1 H + \hat{B}_2 \frac{1}{H} + \hat{B}_3 D + \hat{B}_4 \frac{1}{D}$$

where $\hat{VBAR}$ is predicted VBAR, H is MH in 100-in. sticks to a 7.6-in. top diameter limit, and D is DBH in inches. $\hat{B}_0$ is the sample intercept or regression constant, and $\hat{B}_1$, $\hat{B}_2$, $\hat{B}_3$, and $\hat{B}_4$ are the sample regression coefficients related to the independent variables.

Table 1 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs along with coefficients of variation (R^2) and standard errors of the estimate ($s_{y.x}$) for the height only models. Table 2 shows the cubic-foot and three board-foot VBARs for various MHs based on Equations 1 - 4. Table 2 also shows International 1/4-inch VBARs used by the MDNR (Tally Sheet R 4145). The new International 1/4-inch VBARs are 11.9% smaller and 4.2, 2.9, 3.1, 3.2, and 2.7% larger than the MDNR VBARs for trees with 1, 2, 3, 4, 5, and 6 sticks, respectively. In

Table 1. Estimated intercepts ($\hat{B}_0$), regression coefficients $\hat{B}_1$ and $\hat{B}_2$, and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and three board-foot VBAR prediction equations with height independent variables.

Prediction Equation	$\hat{B}_0$	$\hat{B}_1$	$\hat{B}_2$	R^2	$s_{y \cdot x}$
(1) Cubic-foot[a]	10.897	3.605	-6.767	0.954	1.80
(2) Doyle[b]	-16.117	24.701	9.533	0.948	10.15
(3) International 1/4-inch[c]	35.253	26.090	-24.142	0.959	11.09
(4) Scribner[d]	27.161	26.294	-19.309	0.961	10.56

[a] $\hat{VBAR} = 10.897 + 3.605 \cdot H - 6.767 \cdot \frac{1}{H}$

[b] $\hat{VBAR} = -16.117 + 24.701 \cdot H + 9.533 \cdot \frac{1}{H}$

[c] $\hat{VBAR} = 35.253 + 26.090 H - 24.142 \cdot \frac{1}{H}$

[d] $\hat{VBAR} = 27.161 + 26.294 \cdot H - 19.309 \cdot \frac{1}{H}$

Table 2. VBARs in cu. ft./sq. ft. or bd. ft./sq. ft. for the four types of volumes for various values of MH. International 1/4-inch VBARs are also given from DNR Tally Sheet R 4145.

MH	Cu. ft.	Bd. ft./Sq. ft.			
(sticks)	Sq. ft.	Doyle	Scribner	International 1/4-inch	
				New	DNR
1	7.74	18	34	37	42
2	14.72	38	70	75	72
3	19.46	61	100	105	102
4	23.63	85	128	134	130
5	27.57	109	155	161	156
6	31.40	134	182	188	183
7	35.17	158	208	214	---
8	38.89	183	235	241	---
9	42.59	207	262	267	---
10	46.27	231	288	294	---

general, the new VBARs are in relatively close agreement with the MDNR VBARs.

Table 3 shows the sawtimber VBAR prediction equations for cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBARs for the height and diameter models. Note that R^2 and $s_{y \cdot x}$ for these equations are somewhat larger and smaller, respectively, than for the respective equations based on height only (Table 1). A sawtimber cubic-foot VBAR table based on Equation 5 is shown in Table 4, and Doyle, International 1/4-inch, and Scribner VBAR tables are shown in Tables 5, 6, and 7 based on Equations 6, 7, and 8, respectively.

Predicting One Type of VBAR from Another

Multiple linear regression equations were developed to predict one type of VBAR from another using the 1725 trees in the prediction data set. Equations were developed for predicting cubic-foot VBAR (CR) as a function of Doyle (DR), International 1/4-inch (IR), and Scribner (SR) VBARs, DR as a function of CR, IR, and SR, IR as a function of CR, DR, and SR, and SR as a function of CR, DR, and IR. These equations and their associated R^2 and $s_{y \cdot x}$ values are shown in Table 8.

Validation

The data set used to validate the prediction equations consisted of 432 trees, the remaining approximately 20% of the total of 2157 trees. For each VBAR equation, the average

Table 3. Estimated intercepts ($\hat{B}_0$), regression coefficients $\hat{B}_1$, $\hat{B}_2$, $\hat{B}_3$, and $\hat{B}_4$ and associated values of R^2 and $s_{y \cdot x}$ for the cubic-foot and three board-foot VBAR prediction equations with height and diameter independent variables.

Prediction Equation	$\hat{B}_0$	$\hat{B}_1$	$\hat{B}_2$	$\hat{B}_3$	$\hat{B}_4$	R^2	$s_{y \cdot x}$
(5) Cubic-foot[a]	18.463	4.537	-5.958	-0.759	-23.205	0.978	1.25
(6) Doyle[b]	-28.739	21.324	8.179	2.218	-18.527	0.958	9.08
(7) International 1/4-inch[c]	55.055	29.618	-22.021	-2.556	-26.505	0.966	10.00
(8) Scribner[d]	53.679	28.741	-16.476	-2.232	-107.045	0.965	9.99

[a] $\widehat{VBAR} = 18.463 + 4.537 \cdot H - 5.958 \cdot \frac{1}{H} - 0.759 \cdot D - 23.205 \cdot \frac{1}{D}$

[b] $\widehat{VBAR} = -28.739 + 21.324 \cdot H + 8.179 \cdot \frac{1}{H} + 2.218 \cdot D - 18.527 \cdot \frac{1}{D}$

[c] $\widehat{VBAR} = 55.055 + 29.618 \cdot H - 22.021 \cdot \frac{1}{H} - 2.556 \cdot D - 26.505 \cdot \frac{1}{D}$

[d] $\widehat{VBAR} = 53.679 + 28.741 \cdot H - 16.476 \cdot \frac{1}{H} - 2.232 \cdot D - 107.045 \cdot \frac{1}{D}$

Table 4. VBAR table showing cu. ft./sq. ft. for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 5).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	7.63	15.15	20.68	25.71	30.76				
10	7.13	14.65	20.18	25.21	30.05	34.78			
11	6.58	14.10	19.63	24.66	29.50	34.23			
12	6.00	13.52	19.05	24.08	28.91	33.65	38.33		
13	5.39	12.91	18.44	23.47	28.30	33.04	37.72	42.36	
14	4.76	12.27	17.80	22.84	27.67	32.41	37.09	41.73	
15	4.11	11.63	17.16	22.19	27.02	31.76	36.44	41.08	45.70
16		10.96	16.49	21.53	26.36	31.10	35.78	40.42	45.04
17		10.29	15.82	20.85	25.69	30.42	35.10	39.75	44.37
18		9.61	15.14	20.17	25.01	29.74	34.42	39.06	43.68
19		8.92	14.45	19.48	24.31	29.05	33.73	38.37	42.99
20		8.22	13.75	18.78	23.62	28.35	33.03	37.67	42.29
21			13.04	18.08	22.91	27.65	32.33	36.97	41.59
22			12.34	17.37	22.20	26.94	31.62	36.26	40.88
23			11.62	16.66	21.49	26.23	30.90	35.55	40.17
24			10.91	15.94	20.77	25.51	30.19	34.83	39.45
25			10.18	15.22	20.05	24.79	29.47	34.11	38.73

Table 5. VBAR table showing Doyle bd. ft./sq. ft. for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 6).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	18.7	35.9	55.9	76.5	97.4				
10	21.1	38.3	58.3	78.9	99.8	120.9			
11	23.5	40.7	60.7	81.3	102.2	123.3			
12	25.8	43.1	63.0	83.7	104.6	125.6	146.8		
13	28.2	45.4	65.4	86.0	106.9	128.0	149.1	170.3	
14	30.5	47.7	67.7	88.3	109.2	130.3	151.4	172.6	
15	32.8	50.0	70.0	90.6	111.6	132.6	153.7	174.9	196.1
16		52.3	72.3	92.9	113.8	134.9	156.0	177.2	198.4
17		54.6	74.6	95.2	116.1	137.2	158.3	179.5	200.7
18		56.9	76.9	97.5	118.4	139.5	160.6	181.8	203.0
19		59.2	79.1	99.8	120.7	141.7	162.9	184.0	205.3
20		61.4	81.4	102.0	123.0	144.0	165.1	186.3	207.5
21			83.7	104.3	125.2	146.3	167.4	188.6	209.8
22			85.9	106.6	127.5	148.5	169.7	190.8	212.0
23			88.2	108.8	129.7	150.8	171.9	193.1	214.3
24			90.4	111.1	132.0	153.0	174.2	195.3	216.5
25			92.7	113.3	134.2	155.3	176.4	197.6	218.8

Table 6. VBAR table showing International 1/4-inch bd. ft./sq. ft. for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 7).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	36.7	77.3	110.6	142.1	172.8				
10	34.4	75.1	108.4	139.8	170.5	200.9			
11	32.1	72.8	106.0	137.5	168.2	198.6			
12	29.8	70.4	103.7	135.1	165.9	196.2	226.4		
13	27.4	68.0	101.3	132.8	163.5	193.8	224.0	254.0	
14	25.0	65.6	98.9	130.3	161.1	191.4	221.6	251.6	
15	22.5	63.2	96.5	127.9	158.6	189.0	219.1	249.1	
16		60.7	94.0	125.5	156.2	186.5	216.7	246.7	276.6
17		58.3	91.6	123.0	153.7	184.1	214.2	244.2	274.2
18		55.8	89.1	120.5	151.3	181.6	211.8	241.8	271.7
19		53.3	86.6	118.1	148.8	179.1	209.3	239.3	269.2
20		50.8	84.1	115.6	146.3	176.7	206.8	236.8	266.7
21			81.6	113.1	143.8	174.2	204.3	234.3	264.2
22			79.1	110.6	141.3	171.7	201.8	231.8	261.7
23			76.6	108.1	138.8	169.2	199.3	229.3	259.2
24			74.1	105.6	136.3	166.7	196.8	226.8	256.7
25			71.6	103.1	133.8	164.1	194.3	224.3	254.2

Table 7. VBAR table showing Scribner bd. ft./sq. ft. for various combinations of DBH and MH in sticks to an approximate 7.6" top diameter limit (Equation 8).

DBH (in.)	MH in Sticks								
	1	2	3	4	5	6	7	8	9
9	34.0	70.9	102.4	132.5	162.1				
10	32.9	69.9	101.4	131.5	161.1	190.4			
11	31.7	68.6	100.1	130.2	159.8	189.1			
12	30.2	67.2	98.7	128.8	158.4	187.7	216.8		
13	28.7	65.7	97.2	127.3	156.8	186.1	215.3	244.3	
14	27.0	64.0	95.5	125.6	155.2	184.5	213.6	242.7	
15	25.3	62.3	93.8	123.9	153.5	182.8	211.9	240.9	269.9
16		60.5	92.0	122.1	151.7	181.0	210.1	239.1	268.1
17		58.7	90.2	120.3	149.8	179.1	208.3	237.3	266.3
18		56.8	88.3	118.4	148.0	177.3	206.4	235.4	264.4
19		54.9	86.4	116.5	146.0	175.3	204.5	233.5	262.5
20		52.9	84.4	114.5	144.1	173.4	202.5	231.6	260.5
21			82.4	112.6	142.1	171.4	200.5	229.6	258.5
22			80.4	110.6	140.1	169.4	198.5	227.6	256.5
23			78.4	108.5	138.1	167.4	196.5	225.6	254.5
24			76.4	106.5	136.1	165.4	194.5	223.5	252.5
25			74.3	104.4	134.0	163.3	192.4	221.5	250.4

Table 8. Regression equations for predicting cubic-foot VBARs (CR) and Doyle (DR), International 1/4-inch (IR), and Scribner (SR) board-foot VBARs from the other three types of VBARs.

Regression Equation	R^2	$s_{y \cdot x}$
$\hat{CR} = 3.5600 + 0.2982\ DR - 0.000660\ DR^2$	0.966	2.18
$\hat{CR} = 1.2628 + 0.1848\ IR - 0.000126\ IR^2$	0.992	0.76
$\hat{CR} = 1.5957 + 0.1941\ SR - 0.000160\ SR^2$	0.987	0.95
$\hat{DR} = -8.4172 + 2.6725\ CR + 0.054390\ CR^2$	0.912	13.18
$\hat{DR} = -5.2256 + 0.5378\ IR + 0.001007\ IR^2$	0.957	9.20
$\hat{DR} = -4.3303 + 0.5719\ SR + 0.000977\ SR^2$	0.968	7.96
$\hat{IR} = -5.9448 + 5.2105\ CR + 0.028993\ CR^2$	0.991	5.18
$\hat{IR} = 11.6705 + 1.6920\ DR - 0.002744\ DR^2$	0.964	10.43
$\hat{IR} = 1.7490 + 1.0553\ SR - 0.000164\ SR^2$	0.999	1.50
$\hat{SR} = -7.1585 + 4.9048\ CR + 0.033313\ CR^2$	0.993	6.44
$\hat{SR} = 9.0872 + 1.6197\ DR - 0.002405\ DR^2$	0.973	8.91
$\hat{SR} = -1.6197 + 0.9465\ IR + 0.000148\ IR^2$	0.999	1.49

relative error as a percent ($\overline{RE}$) was calculated where

$$\overline{RE} = \sum_{i=1}^{n} RE_i/n$$

and $RE_i = [(\hat{VBAR}_i - VBAR_i)/VBAR_i] \times 100$, $\hat{VBAR}_i$ and $VBAR_i$ are the predicted and actual VBARs for the i^{th} tree, and n is the number of trees in the validation data set (n = 432). The relative error as a percent for the sum of the predicted VBARs was also calculated where

$$RE_s = [(\sum_{i=1}^{n} \hat{VBAR}_i - \sum_{i=1}^{n} VBAR_i)/\sum_{i=1}^{n} VBAR_i] \times 100$$

and $\sum_{i=1}^{n} \hat{VBAR}_i$ and $\sum_{i=1}^{n} VBAR_i$ are the sum of the predicted and actual VBARs, respectively.

For the validation data set, average DBH = 12.4 in. (range: 7.9 to 22.7), average MH = 3.7 sticks (range: 1-9), average cubic-foot VBAR = 21.75 cu. ft./sq. ft. (range: 7.29 to 39.73), average Doyle board-foot VBAR = 79.14 bd. ft./sq. ft. (range: 15.75 to 191.56), average International 1/4-inch board-foot VBAR = 123.08 bd. ft./sq. ft. (range: 32.87 to 243.35), and average Scribner board-foot VBAR = 117.55 bd. ft./sq. ft. (range = 30.37 to 236.82).

Table 9 shows $\overline{RE}$, range of RE_i, and RE_s for the cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBAR prediction equations based on height only (Equations 1-4) and height and DBH (Equations 5-8) independent variables. $\overline{RE}$ was 1.4% or less and RE_s was 1.07% or less for all VBAR prediction equations. Plots of RE_i versus $VBAR_i$ showed no prediction

Table 9. Average relative error ($\overline{RE}$), range of RE_i, and relative error of the sum of the predicted values (REs) for the four VBAR prediction equations based on height independent variables (Equations 1-4) and height and diameter independent variables (Equations 5-8).

Prediction Equation		$\overline{RE}$	Range of RE_i	RE_s
Cubic-foot	(1)	0.8	-14.0 to 30.2	0.66
	(5)	0.5	-20.2 to 18.1	0.49
Doyle	(2)	1.4	-38.6 to 43.5	0.90
	(6)	1.1	-34.7 to 39.6	1.07
International	(3)	0.8	-21.4 to 30.3	0.76
1/4-inch	(7)	0.6	-26.1 to 24.9	0.65
Scribner	(4)	0.8	-23.7 to 27.8	0.78
	(8)	0.7	-26.4 to 26.3	0.69

problems for the range of VBARs in the validation data set except for smaller $VBAR_i$s. The VBAR prediction equations somewhat over-predict VBAR for cu. ft./sq. ft. values less than 10 and Doyle, International 1/4-inch, and Scribner bd. ft./sq. ft. values less than 40. There was relatively little difference between height only and height and diameter prediction models in terms of accuracy of predictions. However, the height and diameter models were always the most accurate.

Guidelines for Users

The new cubic-foot and Doyle, International 1/4-inch, and Scribner board-foot VBAR prediction equations using DBH and height independent variables were somewhat more accurate than the respective equations using height independent variables (Tables 1, 3, and 9). However, since the differences in accuracy between the two sets of equations were relatively small, the simpler height only VBAR equations are more than adequate for most situations.

The new International 1/4-inch board-foot VBAR equation would yield per acre estimates from 11.9% lower (all trees with one stick) to 4.2% higher (all trees with two sticks) than estimates based on the VBARs presently used by the MDNR. In general, estimates using our VBARs would be no more than 4% higher than estimates using MDNR VBARs except for stands having many trees with only one sawtimber stick.

We recommend the use of VBAR Equations 1-4 for most cruising situations. For these situations where somewhat more accuracy is needed and DBH is already measured for some other purpose(s), Equations 5-8 may be used.

If the user has access to a set of VBAR equations for just one type of volume, VBARs for other types of volume can be predicted from the appropriate equation in Table 8.

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